Exam 2

Math 253, Spring 2021

Name: _____

Tell me what you know: If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down. You will earn more points by writing down one true thing and no nonsense things than you will by writing down one true thing and a bunch of nonsense things.

Show your work: If you use a result from the textbook that we haven't learned in class, tell me where you found it in the textbook. If you do partial fraction decomposition, show how you solved for your constants. This exam is not testing to see if you can get the right answers. It's testing to see if you know calculus.

Check your answers: Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself: If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 52 points on this exam. That means you should budget about 0.9 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder: The only internet resource that you are allowed to use in conjunction with the exam are the notes that I have written and posted to Canvas. If you use any other online resources or if you communicate with other individuals during the exam, you will receive a zero on the exam and you will be reported to the university for academic misconduct.

1. **Vocabulary** (3 points each)

   (a) Fill in the hypothesis of the **Absolute Convergence Test**:

   If $\sum_{n=n_0}^{\infty} a_n$ converges absolutely, then $\sum_{n=n_0}^{\infty} a_n$ converges.

   (b) What is a **p-series**?

   Any series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$

   (c) What does it mean for the series $\sum_{n=n_0}^{\infty} a_n$ to **converge conditionally**?

   $\sum_{n=n_0}^{\infty} |a_n|$ diverges but $\sum_{n=n_0}^{\infty} a_n$ converges.

2. (4 points) Give an example of a series that diverges by the **divergence test**. You don't have to perform the divergence test, just give an example.

$$\sum_{n=1}^{\infty} n$$

3. (4 points) Give an example of a series that converges by the **alternating series test**. You don't have to perform the alternating series test, just give an example.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

4. (4 points) Give an example of a series that converges by the **root test**. You don't have to perform the root test, just give an example.

$$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

5. Consider the series $\sum_{k=2}^{\infty} e^{-k}$.

(a) (3 points) Suppose that c is a constant. Compute $\int_c^{\infty} e^{-x}\,dx$. Your answer should depend on c.

$$\int_c^{\infty} e^{-x}\,dx = \lim_{t\to\infty} \int_c^t e^{-x}dx = \lim_{t\to\infty} -e^{-x}\Big|_c^t$$

$$= \lim_{t\to\infty} -e^{-t} + e^{-c} = e^{-c}$$

(b) (6 points) Apply the **integral test** to show that $\sum_{k=2}^{\infty} e^{-k}$ converges. You may use your answer from part (a).

$f(x) = e^{-x} > 0$ ✓

$f(x)$ cts. ✓

$f(x)$ decreasing ✓

$f(k) = e^{-k}$ ✓

$\int_2^{\infty} e^{-x}dx = e^{-2} < \infty$, so by the integral test, $\sum_{k=2}^{\infty} e^{-k}$ converges.

(c) (6 points) Note that this is still problem 5. Find an integer, n, so that the nth partial sum, S_n, is within 10^{-100} of the actual sum. You may use your answer from part (a). *You don't have to compute* S_n.

Note: $R_n \leq \int_n^\infty e^{-x}\,dx = e^{-n}$

Goal: $e^{-n} < 10^{-100}$

$\rightarrow -n < \ln(10^{-100})$

$\rightarrow n > -\ln(10^{-100}) \approx 230.2$

Take $\boxed{n = 231}$

6. (8 points) Does the series $\sum_{n=0}^{\infty} \frac{5^n (n!)^2}{(2n)!}$ converge or diverge?

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{5^{n+1}\left((n+1)!\right)^2}{(2(n+1))!} \cdot \frac{(2n)!}{5^n (n!)^2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{5^{n+1}}{5^n} \cdot \frac{(2n)!}{(2n+2)!} \cdot \left(\frac{(n+1)!}{n!} \right)^2 \right|$$

$$= \lim_{n\to\infty} \left| 5 \cdot \frac{(2n)(2n-1)\cdots 2\cdot 1}{(2n+2)(2n+1)(2n)\cdot(2n-1)\cdots 2\cdot 1} \cdot (n+1)^2 \right|$$

$$= \lim_{n\to\infty} \left| \frac{5(n^2+2n+1)}{(4n^2+6n+2)} \right| = \frac{5}{4} > 1$$

By the ratio test, $\sum_{n=0}^{\infty} \frac{5^n (n!)^2}{(2n)!}$ diverges

7. (8 points) Does the series $\sum_{k=1}^{\infty} \frac{1}{4k-3}$ converge or diverge?

$\frac{1}{4k-3} \geq \frac{1}{4k}$

$\sum_{k=1}^{\infty} \frac{1}{4k} = \frac{1}{4} \sum_{k=1}^{\infty} \frac{1}{k}$ diverges by the p-test ($p=1$).

By the D.C.T. $\sum_{k=1}^{\infty} \frac{1}{4k-3}$ diverges.